

## 6.1 The Law of Sines

### Oblique Triangle

An oblique triangle is a triangle that does not contain a right angle.

An oblique triangle has either three acute angles or two acute angles and one obtuse angle. The angles are labeled A, B, and C. The sides opposite each angle are labeled as a, b, and c, respectively.

### The Law of Sines

If ABC is a triangle with sides a, b, and c ; angles A, B, and C, then the ratio of the length of the side of any triangle to the sine of the angle opposite that side is the same for all three sides of the triangle. That is,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ .

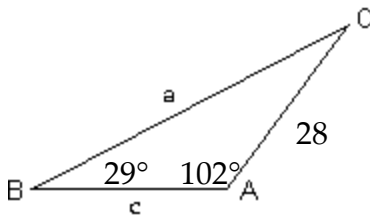
### Solving Oblique Triangles

Solving an oblique triangle means finding the measurements of its angles and the lengths of its sides. The Law of Sines can be used if a side and two angles are known (SAA) or two angles and the side between them are known (ASA).

**Solve the triangle. Round lengths and angle measures to the nearest tenth.**

A = 102°, B = 29°, and b = 28 feet (One Side and Two Angles - SAA Case)

1)



1) \_\_\_\_\_

$$\angle C = 180^\circ - (29^\circ + 102^\circ) = 49^\circ$$

$$\frac{a}{\sin 102^\circ} = \frac{28}{\sin 29^\circ}$$

$$\rightarrow a \cdot \sin 29^\circ = 28 \cdot \sin 102^\circ \rightarrow a = \frac{28 \cdot \sin 102^\circ}{\sin 29^\circ} \approx 56.5 \text{ ft.}$$

$$\frac{c}{\sin 49^\circ} = \frac{28}{\sin 29^\circ}$$

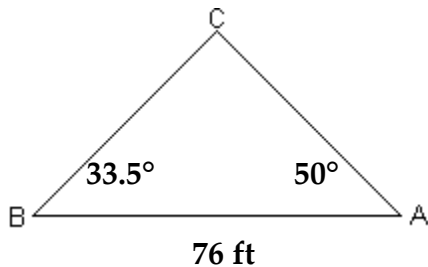
$$\rightarrow c \cdot \sin 29^\circ = 28 \cdot \sin 49^\circ \rightarrow c = \frac{28 \cdot \sin 49^\circ}{\sin 29^\circ} \approx 43.6 \text{ ft.}$$

Solve the triangle. Round lengths and angle measures to the nearest tenth.

A = 50°, B = 33.5°, c = 76 feet (Two Angles and the Side Between Them - ASA Case)

2)

2) \_\_\_\_\_



$$\angle C = 180^\circ - (33.5^\circ + 50^\circ) = 96.5^\circ$$

$$\frac{a}{\sin A} = \frac{c}{\sin C} \rightarrow \frac{a}{\sin 50^\circ} = \frac{76}{\sin 96.5^\circ}$$

$$\rightarrow a \cdot \sin 96.5^\circ = 76 \cdot \sin 50^\circ \rightarrow a = \frac{76 \cdot \sin 50^\circ}{\sin 96.5^\circ} \approx 58.6 \text{ ft.}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \frac{b}{\sin 33.5^\circ} = \frac{76}{\sin 96.5^\circ}$$

$$\rightarrow b \cdot \sin 96.5^\circ = 76 \cdot \sin 33.5^\circ \rightarrow b = \frac{76 \cdot \sin 33.5^\circ}{\sin 96.5^\circ} \approx 42.2 \text{ ft.}$$

### Area of an Oblique Triangle

The area of a triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,  $\text{Area} = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C$ .

Find the area of triangle ABC. Round to the nearest square unit.

3) C = 135°, a = 90 m, b = 50 m

3) \_\_\_\_\_

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} (90)(50) (\sin 135^\circ) \approx 1591 \text{ sq m.}$$

Solve the problem. Round to the nearest tenth.

4) A pole leans at an angle of 82°, and it casts a 22-foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is 43°. Find the length of the pole.

4) \_\_\_\_\_

$$\frac{x}{\sin 43^\circ} = \frac{22}{\sin 55^\circ}$$

$$\rightarrow x \cdot \sin 55^\circ = 22 \cdot \sin 43^\circ \rightarrow x = \frac{22 \cdot \sin 43^\circ}{\sin 55^\circ} \approx 18.3 \text{ ft.}$$

- 5) Two fire-lookout stations are 20 miles apart, with station B directly east of station A. Both stations spot a fire on a mountain to the north. The bearing from station A to the fire is N 50°E. The bearing from station B to the fire is N 36°W. How far is the fire from station A. 5) \_\_\_\_\_

$$\frac{x}{\sin 54^\circ} = \frac{20}{\sin 86^\circ}$$

$$\rightarrow x \cdot \sin 86^\circ = 20 \cdot \sin 54^\circ \rightarrow x = \frac{20 \cdot \sin 54^\circ}{\sin 86^\circ} \approx \mathbf{16.2 \text{ miles.}}$$

- 6) A plane flies 160 miles with a bearing of 28° from airport A to airport B. Then it flies due east ( bearing 90°) from airport B to airport C. The course to return to airport A had a bearing of 219°. Find the distance from airport A to airport C. 6) \_\_\_\_\_

$$\frac{x}{\sin 118^\circ} = \frac{160}{\sin 51^\circ}$$

$$\rightarrow x \cdot \sin 51^\circ = 160 \cdot \sin 118^\circ \rightarrow x = \frac{160 \cdot \sin 118^\circ}{\sin 51^\circ} \approx \mathbf{181.8 \text{ mi.}}$$

6.1 Exercises pg 690 (3, 7, 33, 43, 52) (2, 4, 37, 44, 54)

## 6.2 The Law of Cosines

### Law of Cosines

The Law of Cosines can be used to solve a triangle in which two sides and their included angle are known (SAS), or a triangle in which three sides are known (SSS).

#### Standard Form

$$a^2 = b^2 + c^2 - 2bc \cos A$$

#### Alternative Form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

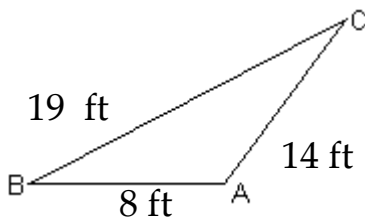
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

### Solving a SSS Triangle

1. Find the angle opposite the longest side.
2. Find either of the two remaining acute angles.
3. Find the third angle.

Solve the triangle. Round angle measures to the nearest tenth.

7)



7) \_\_\_\_\_

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{14^2 + 8^2 - 19^2}{2(14)(8)} \approx -0.4509$$

$$\rightarrow \angle A = \cos^{-1}(-0.4509) \approx 116.8^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{19^2 + 8^2 - 14^2}{2(19)(8)} \approx 0.7533$$

$$\rightarrow \angle B = \cos^{-1}(0.7533) \approx 41.1^\circ$$

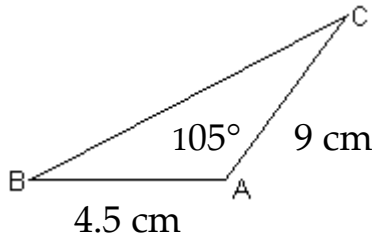
$$\text{So, } \angle C = 180^\circ - (116.8^\circ + 41.1^\circ) \approx 22.1^\circ$$

## Solving a SAS Triangle

1. Find the side opposite the given angle.
2. Find the angle opposite the shorter of the two given sides.  
This angle is always acute.
3. Find the third angle.

Solve the triangle. Round lengths of sides and angle measures to the nearest tenth.

8)



8) \_\_\_\_\_

1.  $a^2 = b^2 + c^2 - 2bc \cos A$   
 $= 9^2 + 4.5^2 - 2(9)(4.5) \cos 105^\circ \approx 122.2$   
 $\rightarrow a = \sqrt{122.2} \approx 11.1 \text{ cm}$
2.  $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{11.1^2 + 9^2 - 4.5^2}{2(11.1)(9)} \approx 0.9207$   
 $\rightarrow \angle C = \cos^{-1}(0.9207) \approx 23.0^\circ$
3.  $\angle B = 180^\circ - (105^\circ + 23^\circ) \approx 52.0^\circ$

Solve the problem. Round to the nearest unit.

- 9) A ship leaves port A and travels 740 miles to port B on a bearing of N57°E. The ship later leaves port B to port C 520 miles away on a bearing of S63°E. Find the distance from port A to port C.

9) \_\_\_\_\_

$$x^2 = 740^2 + 520^2 - 2(740)(520)(\cos 120^\circ) \approx 1,202,800$$
$$\rightarrow x = \sqrt{1,202,800} \approx 1097 \text{ miles.}$$

- 10) Two airplanes leave an airport at the same time on different runways. One flies on a bearing of N66°W at 325 mph. The other airplane flies on a bearing of S26°W at 300 mph. How far apart will the airplanes be after 2 hours?

10) \_\_\_\_\_

$$x^2 = 650^2 + 600^2 - 2(650)(600)(\cos 88^\circ) \approx 755,278$$
$$\rightarrow x = \sqrt{755,278} \approx 869 \text{ miles.}$$

## Heron's Area Formula

The area of a triangle with sides  $a$ ,  $b$ , and  $c$  is  $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$  ;  
where  $s$  is one-half its perimeter  $= \frac{a+b+c}{2}$ .

Find the area of the triangle using Heron's formula. Round to the nearest square unit.

11)  $a = 43 \text{ m}$  ,  $b = 56 \text{ m}$  ,  $c = 72 \text{ m}$  11) \_\_\_\_\_

$$s = \frac{a+b+c}{2} = \frac{43+56+72}{2} = 85.5 \text{ m}$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(85.5)(85.5-43)(85.5-56)(85.5-72)} \\ &= \sqrt{(85.5)(42.5)(29.5)(13.5)} \approx 1203 \text{ sq m}\end{aligned}$$

6.2 Exercises pg 699 (3, 7, 27, 39) (11, 17, 51, 40)